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Scheme to Improve Limit Cycle Performance of an Attitude Control System

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FIGURE 1 is a block diagram of a single axis attitude control system in which an attitude signal, θ^*, is derived by integrating the signal from a precision rate gyro. A stabilizing signal ϵ^* is also derived in the computer

$$\epsilon^* = K_R\omega^* + K_P\theta^*$$

where

$$\theta^* = \int \omega^* dt$$

ϵ^* is used to determine the times at which the thruster is fired, the direction of the thrust, and the duration of the thrust pulse. These three items of intelligence are represented as the valve command, τ_c, in the diagram. The thruster is characterized by the strength of the thrust, A (assumed constant during the pulse), a time delay τ_D between the instant a turn-on command is received and the instant thrust actually occurs, and a minimum time, τ_o, which the pulse width modulator may command as thruster on-time. ω is the angular rate of the vehicle about its controlled axis and is related to the thrust M in the usual way: $M = J\dot{\omega}$. The gyro output signal m_g is dynamically related to the vehicle rate by

$$\ddot{\omega}_g + (2\zeta\omega_n)\dot{\omega}_g + (\omega_n^2)\omega_g = (\omega_n^2)\omega$$

where ζ is the damping ratio and ω_n the undamped natural frequency of the output axis of the gyro. T is the cycle time of the computer, and the analog to digital conversion is represented by a zero order hold, the output of which is ω^*_g, a piecewise constant representation of ω_g.

A conventional method of control for this system is to program the computer to issue a thruster command at each sample instant. The pulse width command at instant t_k is determined from $\epsilon^*(t_k)$ according to the curve in Fig. 2. A dead zone of total width $2W$ is employed to prevent the valve from responding to noise which might exist on the ϵ^* signal. The parameters W, S, K_R, K_P, T, ζ, and ω_n may then all be adjusted to provide acceptable performance of the system where τ_D, τ_o, A, and J are considered fixed.

Insofar as limit cycle performance is concerned these seven parameters are adjusted to minimize control fuel expenditure (total valve on-time) while maintaining acceptable limits on the excursions of ω and θ during the limit cycle oscillation. The conventional control scheme described above leads to a certain optimum limit cycle performance represented qualitatively by the $\epsilon^*(t)$ signal in Fig. 3. Here that portion of the limit cycle during which $\epsilon^*(t)$ reaches its peak is shown. Prior to t_o the vehicle is drifting at a constant rate. t_o is the sample instant at which $\epsilon^*(t)$ first exceeds the dead-zone value W. A pulse width is calculated from Fig. 2 and a thruster command is issued as soon as this calculation is completed. There is a delay, τ_D, between the initiation of the thruster command and the actual occurance of thrust. Also, the gyro does not

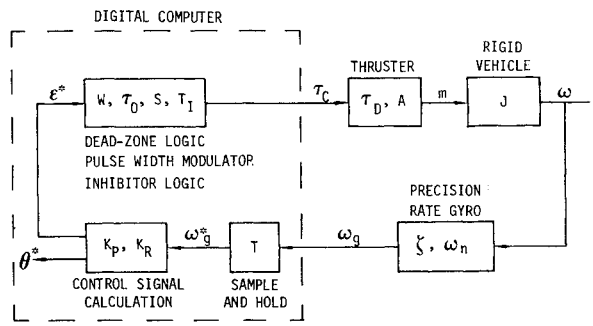

Fig. 1 Single axis attitude control system.

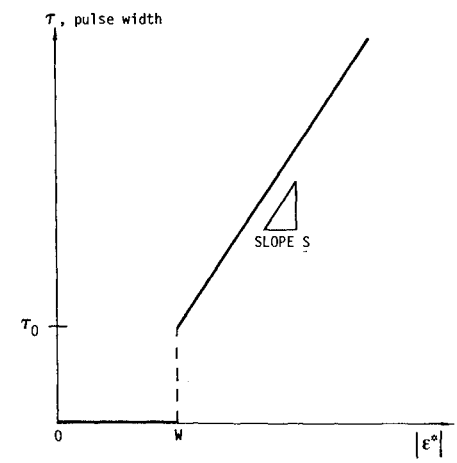

Fig. 2 Pulse-width modulation.

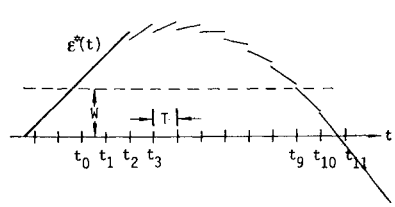

Fig. 3 $\epsilon^*(t)$ vs t during its reversal.

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respond instantly to vehicle motions. Consequently the $\epsilon^*(t)$ signal continues to increase at a constant rate until t_1. At instant t_1, a new pulse width is calculated which will be longer than that calculated at t_o, and this now becomes the thruster command. Assuming that τ_o is longer than T, this simply means that at t_1 the thruster command is continued. Because of the thruster delay, the gyro dynamics, and the sample and hold operation, the $\epsilon^*(t)$ signal will remain unaffected by the vehicle acceleration caused by the thruster for some time after t_1. The thruster will actually come on at time $t_o+\tau_D$, which would be sometime around t_1, if T and τ_D are approximately the same. Here we show $\epsilon^*(t)$ increasing at its constant rate until t_2, the sample instant at which the effect of the thruster is first sensed at the pulse width modulator. $\epsilon^*(t_2)$ is even larger than $\epsilon^*(t_1)$, however, so the pulse width calculated at t_2 will be longer than that at t_1, and the thrust command will be continued. $\epsilon^*(t)$ will therefore remain above the dead-zone level for several sample periods. Here it drops below W for the first time at t_{10}, so at t_{10} the thrust command is set to zero. At some later instant, shown at t_{11} here, $\epsilon^*(t)$ will be in the dead-zone and decreasing at a constant rate.

The thruster pulse duration during this reversal in $\epsilon^*(t)$ is much longer than the minimum thruster on time because of the time delays in the control loop. Therefore, the vehicle rate during the limit cycle oscillation is higher than the minimum which is physically possible, so the limit cycle is less efficient than is physically possible.

A new control law, designed to overcome the effects of the time lag and improve the limit cycle performance, has been devised. This scheme utilizes the logical capabilities of the digital computer in addition to its computational abilities, and is called the inhibitor model of control.[1]

The basic inhibitor mode works during the limit cycle in the following way. Referring to Fig. 3, at t_o the pulse width is calculated and the thruster command issued in the same manner as in the conventional scheme described. In the case shown here, this pulse width will be slightly larger than the minimum τ_o because $\epsilon^*(t_o)$ happens to be slightly greater than W. At the time the pulse command is issued the computer switches into the "inhibit" mode, which simply means that $\epsilon^*(t)$ will be ignored (assumed to be zero) for a given period of time—on the order of a few cycle periods. The pulse which is commanded at t_0 will be emitted, the vehicle will decelerate and reverse direction, and the gyro will react and cause $\epsilon^*(t)$ to reverse direction and even drop below the threshold level W during the period of inhibition. When the inhibit period is completed, the computer reverts to its alert mode and will command a corrective pulse the next time ϵ^* exceeds the threshold level.

To investigate the feasibility of the inhibitor scheme, a simulation of the system shown in Fig. 1 was established. For this simulation, the parameter values shown in Table 1 were used. T_I is the inhibit period. In each case reported here, a two-pulse limit cycle, symmetrical in ω, was established. The results of these tests are summarized in Fig. 4 where the pulse duration which resulted in each case is plotted against the K_R gain used. In the conventional case the limit cycle was unstable (or did not exist) for $K_R<0.5$.

Table 1 Parameter values used in simulation

τ_D	=1
A	=0.02
J	=1
ω_n	=1.2
ζ	=0.7
T	=1
K_P	=0.2
K_R	=variable, see Fig. 4
W	=1
τ_0	=1.5
S	=1
T_I	=0 for conventional control law
T_i	=5 for inhibitor mode

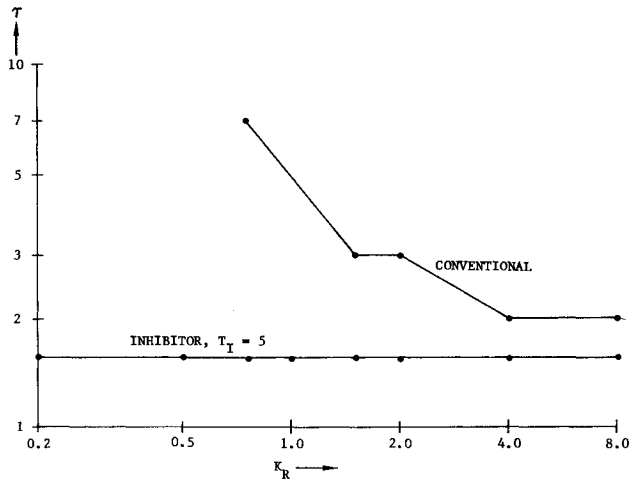

Fig. 4 Pulse-width vs K_R for $K_P=0.2$.

For a symmetrical limit cycle, the important performance parameters can be established from τ. In most attitude control systems the positional accuracy is essentially determined by the dead-zone width W, so is independent of τ. The long-term fuel expenditure is proportional to τ^2, so if this is taken as the performance measure, we see that the inhibitor scheme, in the system tested here, gave an improvement in performance over the conventional control law ranging from 20.4 to 1.7, depending on K_R.

There are many possible variations of the basic inhibitor scheme which is described here. Stability of two pulse limit cycles and the existence of higher order limit cycles are important considerations in any extensions of the inhibitor idea. In the presence of disturbance torques, a situation not investigated here, it would probably be necessary to provide for a variable inhibit period T_I to prevent excessive excursions of ω.

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Velocity of Bodies Powered by Rapidly Discharged Cold-Gas Thrusters

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Nomenclature

c_i	= initial sound speed	(m/sec)
I_s	= specific impulse	(m/sec)
k	= specific-heats parameter, $k=2/(\gamma-1)$	
m_f	= final vehicle mass	(kg)

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